A STUDY OF DISCONTINUITY SURFACES WITH ENERGY RELEASE (OR ABSORPTION) IN MAGNETOHYDRODYNAMICS

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We consider discontinuity surfaces, at which exothermic or endothermic reactions occur, in electrically-conducting media with infinite conductivity in the presence of an arbitrary magnetic field.

The study of discontinuities with exothermic reactions in conducting media, when the flow velocity is perpendicular to the surface and the magnetic field parallel to it, has been carried out numerically by Gross, Chinitz and Rivlin [1]. We observe that this case may be reduced to gas dynamics, with a different relation between the internal energy and density.

Demutskii and Polovin [2] studied discontinuities with endothermic and exothermic reactions for arbitrary orientations of the field and velocity, but for the case where the energy released and the square of the Alfven speed are much smaller than the square of the sound speed.

Below, we consider discontinuities with release (or absorption) of energy, without restricting the magnitude and direction of the velocity, the direction of the magnetic field, and the amount of energy release. It will be shown that in the general case, there occur two types of detonations and two types of ionization shocks, in which the regions of reaction are behind the fast shocks and slow shocks respectively. Moreover, there may occur four types of combustion fronts, two of which are compression waves, the other two expansion waves. The investigation will give the changes (across the shocks) in density, pressure, magnetic field, gas velocity, temperature, and entropy as functions of strength of the wave and the energy released.

Limits are obtained on the magnitude of the field and the amount of the energy released, at which no longer occur slow detonations and two types of combustion, the speed of which may be close to the Alfven speed. Some partial results of this paper have been published in [3].

1. We shall consider media, in which the internal energy is related to the pressure p and density ρ thus:

$$e = \frac{p}{(\gamma - 1)\rho} + C \qquad \left(T = \rho^{\gamma-1} f\,(p\rho^{\gamma-1})\right)$$

We assume that γ and C change at the discontinuity, and that the temperature $T = T(p, \rho)$ has the above form, as given in [4]; i.e. we do not require the medium to be a perfect gas. The nature of the discontinuity, and the changes in all the other variables considered, with the exception of temperature and entropy, do not depend on the form of the function f. Thus, we shall assume for simplicity that $p = R_0 \rho T$.

In order for the reaction to continue, there must be an in-flow of reacting substances. Consequently, we shall consider surfaces across which the mass flow is nonzero. In the case where the density is continuous, the basic jump relations assume the form [5]

$$p_2 - p_1 = \frac{\gamma_2 - \gamma_1}{\gamma_1 - 1} + (C_1 - C_2)(\gamma_2 - 1)\rho_1, \qquad H_{t2}^2 - H_{t1}^2 = 8\pi (p_2 - p_1) \quad (1.1)$$

$$v_{n1} = v_{n2} = \frac{H_n}{\sqrt{4\pi\rho_1}}, \qquad \mathbf{v}_{t2} - \mathbf{v}_{t1} = \frac{1}{\sqrt{4\pi\rho_1}} (\mathbf{H}_{t2} - \mathbf{H}_{t1}) \quad (1.2)$$

Here, and in what follows, the subscripts *1* and *2* denote quantities in front of and behind the discontinuity, respectively. H_n, v_n, $\mathbf{H}_t$, $\mathbf{v}_t$ denote respectively the normal and tangential components (to the surface) of the magnetic field and gas velocity relative to the surface.

From this, it is clear that the pressure, magnitude of the tangential component of the magnetic field, and the velocity, all change across the shock discontinuously, the changes being determined by the initial state and the amount of energy release. Moreover, the magnetic field and velocity may be rotated through an arbitrary angle about the normal to the discontinuity surface.

2. We shall now consider that the density jumps. It is readily shown [5] that in this case the magnetic field strengths and velocities on both sides of the discontinuity surface lie in one plane. This implies that the projections of these vectors in the plane of the discontinuity are colinear. We introduce two dimensionless parameters

$$\eta = \frac{H_{t2}}{H_{t1}} - 1, \qquad \xi = -h_1 \frac{\Delta u_0}{\Delta v_0} \qquad \left(h_1 = \frac{H_{t1}}{H_{t2}}\right) \quad (2.1)$$

Here Δu_0, Δv_0 are the jumps in the normal and tangential components of the gas velocity. Using the laws of conservation of mass and momentum, and using the continuity of the tangential component of the electric field strength $\mathbf{E} = (\mathbf{v} \times \mathbf{H})/c$, we can express the jumps in all the quantities and gas velocities relative to the shock in terms of these parameters and h_1

$$\begin{gathered}\Delta R = R_2 - 1 = \frac{\xi\eta}{\xi+1}, \qquad \Delta P = P_2 - P_1 = \eta\,[\xi - h_1^2(\eta/2+1)] \\ \Delta P^* = P_2^* - P_1^* = \eta\xi, \quad V_1^2 = (\eta+1)\,\xi + 1, \quad V_2^2 = \xi + 1 \\ \Delta u = \frac{\eta\xi}{\sqrt{(\eta+1)\,\xi+1}}, \quad \Delta v = -\frac{\eta h_1}{\sqrt{(\eta+1)\,\xi+1}}, \quad \Delta h = \eta h_1\end{gathered} \tag{2.2}$$

Here P_i, R_2, P_i^* are the nondimensional pressure, density, and total pressure, respectively, V_i the nondimensional gas speed relative to the discontinuity, and Δu, Δv the nondimensional jumps in the components of the gas velocities, thus:

$$\begin{gathered}P_i = \frac{4\pi p_i}{H_n^2}, \qquad R_2 = \frac{\rho_2}{\rho_1}, \qquad P_i^* = P_i + \frac{1}{2}\frac{H_i^2}{H_n^2} \\ V_i = \frac{D_i}{H_n/\sqrt{4\pi\rho_i}}, \qquad \Delta u = \frac{\Delta u_0}{H_n/\sqrt{4\pi\rho_1}}, \qquad \Delta v = \frac{\Delta v_0}{H_n/\sqrt{4\pi\rho_1}}\end{gathered} \tag{2.3}$$

Using the equation of state of Clapeyron, we obtain, for the temperature jump:

$$\Delta\Theta = \Theta_2 - \Theta_1 = \frac{\eta\,(\xi+1)\,[\xi - \frac{1}{2}h_1^2(\eta+2)] - P_1\xi\eta}{(\eta+1)\,\xi+1} \quad \left(\Theta_i = \frac{R_{0i}T_i}{H_n^2/4\pi\rho_1},\ i = 1, 2\right) \tag{2.4}$$

The law of conservation of energy then permits us to relate $\xi = \xi(\eta, q)$ to the parameters before the front; we have

$$\begin{gathered}f(\eta, \xi;\, h_1^2, P_1, q, \gamma_2) \equiv \xi^2\eta\,[2 - (\gamma_2 - 1)\,\eta] + \\ + 2\xi\left[\frac{\gamma_2-2}{2}h_1^2\eta^2 - (h_1^2 + \gamma_2 P_1 + q - 1)\,\eta - q\right] - \eta(\eta+2)h_1^2 - 2q = 0\end{gathered} \tag{2.5}$$

where

$$q_2 = (\gamma_2 - 1)\frac{e_1(p_1, \rho_1) - e_2(p_1, \rho_1)}{H_n^2/4\pi\rho_1} = (\gamma_2 - 1)\left[\frac{\gamma_2 - \gamma_1}{(\gamma_2 - 1)(\gamma_1 - 1)}P_1 + \frac{C_1 - C_2}{H_n^2/4\pi\rho_1}\right]$$

Here $C_1 - C_2$ is the chemical energy released per unit mass. In exothermic reactions $q > 0$, in endothermic reactions $q < 0$.

To compute the entropy change in the state behind the shock in terms of the shock parameters, we use the identity

$$TdS_0 = \frac{1}{\gamma - 1} d\frac{p}{\rho} + pd\frac{1}{\rho}$$

This implies that the entropy change has been assumed to be independent of the change in the concentration of the substances.

Using the law of conservation of energy at the discontinuity, the above identity may be transformed to

$$\Theta_2 dS_2 = \frac{dq}{\gamma_2 - 1} + \frac{1}{2}\frac{d\xi_0}{(\xi_0+1)^2}\eta^2(\xi^2 + h_1^2) \qquad \left(S_2 = \frac{S_0}{R_{02}},\ \xi_0 = (\eta+1)\xi\right) \tag{2.6}$$

We shall prescribe the thermodynamic state and the magnetic field in front of the discontinuity, the energy released q, γ_2, and either one of the two shock strength parameters ξ, η, or a relation between them $\psi(\xi, \eta) = 0$. Then (2.5) and the indicated relation will serve to find η and ξ as functions of the prescribed quantities, and relations (2.2) and (2.4) will give all the quantities behind the shock as well as its velocity.

We now carry out a qualitative study of Equation (2.5) in the $\eta\xi$-plane as a function of the parameters P_1, h_1^2, q. Since $P_2 \geqslant 0$, $V_1^2 \geqslant 0$, $V_2^2 \geqslant 0$, it follows from (2.2) that only the portion of the plane given by the following inequalities has physical meaning:

$$\eta\left[\xi - \tfrac{1}{2}h_1^2(\eta+2) + P_1/\eta\right] \geqslant 0, \qquad (\eta+1)\xi + 1 \geqslant 0, \qquad \xi + 1 \geqslant 0$$

In Fig. 1, the equal signs correspond to the curves *1, 7, 8*. We note that the lines of constant mass-flows across the shock are given by the hyperbolas $(\eta + 1)\xi$ = const (analogous to the rays in the $p\rho^{-1}$-plane in ordinary gas dynamics). Different points on the axis $\eta = 0$ for $\xi > -1$ correspond to the states in front of the shock with different values of mass flow.

We shall go into the special case $q = 0$. Relation (2.5) assumes the form

$$\eta\{\eta\,[(\gamma_2 - 1)\,\xi^2 - (\gamma_2 - 2)\,h_1^2\xi + h_1^2] - 2\,[\xi^2 - (h_1^2 + \gamma_2 P_1 - 1)\,\xi - h^2]\} = 0$$

The case $\eta = 0$ corresponds to continuous flow, and the bracket going to zero corresponds to magnetohydrodynamic shocks.

The function

$$\eta = 2\,\frac{\xi^2 - (h_1^2 + \gamma_2 P_1 - 1)\,\xi - h_1^2}{(\gamma_2 - 1)\,\xi^2 - (\gamma_2 - 2)\,h_1^2\xi + h_1^2} \qquad (\xi = \xi(\eta, 0)) \tag{2.7}$$

for different values of h_1^2, P_1 and γ_2 is represented in Fig. 2; the curves *1* and *2* correspond to the case $(\gamma_2 - 1)h_1^2 > 1 - \gamma_2 P_1$, curve *3* to $1 - \gamma_2 P_1 > (\gamma_2 - 1)h_1^2 > (\gamma_2 - 1)^2(1 - \gamma_2 P_1)$, and curve *4* to $(\gamma_2 - 1)(1 - \gamma_2 P_1) > h_1^2$.

From the relation (2.6), it is clear that the entropy rises on the parts of the curves shown as full lines in Fig. 2. According to (2.2), these parts of the curves correspond to compression discontinuities ($\Delta P > 0$, $\Delta R > 0$); moreover, it is easily seen that the upper parts correspond to fast magnetohydrodynamic shock waves, and the lower parts to slow waves.

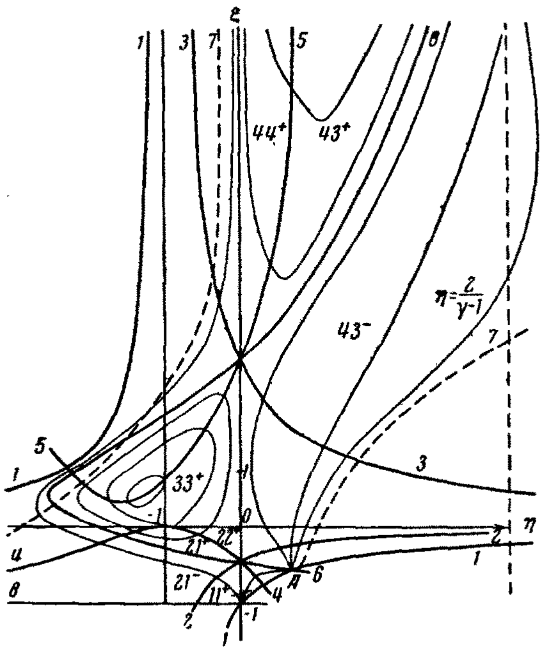

Fig. 1.

In Fig. 1, the fine lines are curves $\xi = \xi(\eta;\ h_1^2,\ P_1,\ q)$ for fixed h_1^2, P_1, γ_2 and different q. Curve *6* corresponds to $q = 0$. According to (2.5), all the curves in this family intersect at points lying on the hyperbola $(\eta + 1)\xi + 1 = 0$. These points are $A(\xi_a,\ \eta_a)$ and $B(\xi_b,\ \eta_b)$, and in the case corresponding to curve *1* in Fig. 2, also points $C(\xi_c,\ \eta_c)$ and $D(\xi_d,\ \eta_d)$. Here

$$\xi_a = -1, \quad \eta_a = 0$$

$$-1 < \xi_b < 0, \quad \eta_b > 0$$

$$\xi_c, \xi_d > 0, \quad \eta_c,\ \eta_d < -1$$

Furthermore, it follows from (2.5) that for small values $|q|$ the family of curves $\xi = \xi(\eta,\ q)$ differ but slightly from the curve $\xi = \xi(\eta,\ 0)$, and for large values of $|q|$ but bounded values of η and ξ, they are close to the hyperbola

$$\xi(\eta + 1) + 1 = 0$$

We also note that the curves intersect the axis $\xi = 0$ at the points $\eta = -1 \pm \sqrt{(1 - 2q/h_1^2)}$ and have vertical asymptotes $\eta = 0$, $\eta = 2/(\gamma_2 - 1)$, near which the following expansions are valid

$$\xi = \frac{q}{\eta} + h_1^2 + \gamma_2 P_1 + \frac{\gamma_2 + 1}{2} q + O(\eta) \qquad \text{for } |\eta| \ll 1 \tag{2.8}$$

$$\xi = -\frac{2}{\eta - 2/(\gamma_2 - 1)} \left[h_1^2 + (\gamma_2 - 1)(\gamma_2 P_1 - 1) + \frac{1}{2}(\gamma_2^2 - 1) q \right] + O(1) \quad \text{for } |\eta - 2/(\gamma_2 - 1)| \ll 1$$

In addition, straight lines parallel to either the η-axis or the ξ-axis intersect the curves at no more than two points.

3. We shall consider, as in ordinary gas dynamics, the two types of discontinuities.

1. Discontinuities whose velocities depend on the wave strengths and are determined by the solutions of the magnetogasdynamic problems: These discontinuities are the shock waves, detonation waves (i.e. shock waves followed immediately by a reaction zone), ionization waves (i.e. shock waves accompanied by ionization). We shall call these discontinuities the "detonation" type. The conditions of evolutionarity, i.e. stability relative to splitting (or resolution) of these discontinuities, are given by the inequalities [2,3]

$$V_1^2 > a_{1+}^2, \qquad 1 < V_2^2 \leqslant a_{2+}^2$$

or

$$1 > V_1^2 > a_{1-}^2, \quad V_2^2 \leqslant a_{2-}^2 \tag{3.1}$$

where a_{i+}, a_{i-} are the dimensionless velocities of the fast and slow magnetosonic waves.

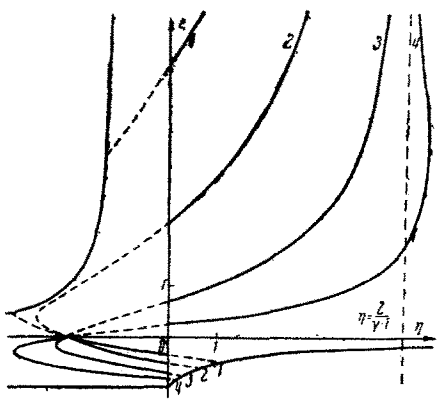

Fig. 2.

2. Discontinuities whose velocities do not depend on the wave strengths and are determined by the physico-chemical characteristics of the media.

These discontinuities are the ordinary or thermonuclear combustion fronts, condensation shocks, photo-ionization discontinuities. We shall call these discontinuities the "combustion" type. The evolutionarity of these discontinuities are determined by one of the following pairs of inequalities

$$V_1^2 < a_{1-}^2, \quad V_2^2 \leqslant a_{2-}^2; \qquad a_{1-}^2 < V_1^2 < 1, \quad a_{2-}^2 \leqslant V_2^2 < 1$$
$$1 < V_1^2 < a_{1+}^2, \quad 1 < V_2^2 \leqslant a_{2+}^2; \qquad a_{1+}^2 < V_1^2, \quad a_{2+}^2 \leqslant V_2^2 \tag{3.2}$$

To investigate the evolutionarity of the discontinuities, we construct curves in the $\eta\xi$-plane, along which the gas-speed relative to a discontinuity before and after it equal the speeds of small magnetohydrodynamic disturbances. In Fig. 2, the equalities $V_1^2 = a_{1+}^2$, $V_1^2 = a_{1-}^2$ $V_1^2 = 1$ correspond to the hyperbolas *2* and *3* and the pair of intersecting straight lines $\xi = 0$, $\eta = -1$. Equation $V_2^2 = 1$ corresponds to the straight line $\xi = 0$, and equations $V_2^2 = a_{2+}^2$ and $V_2^2 = a_{2-}^2$ to the Jouguet curves *4* and *5*. Equations of the lines 2-5 follow:

$$(\eta + 1)\,\xi + 1 = a_{1+}^2, \qquad (\eta + 1)\,\xi + 1 = a_{1-}^2$$

$$\varphi(\eta, \xi) \equiv (\eta + 1)^2 h_1^2 \left(1 - \frac{\gamma_2 - 1}{2}\,\xi\right) + \gamma_2 (\eta + 1)\,\xi^2 - (\gamma_2 + 1)\,\xi^2 + \\ + \xi\left(\frac{\gamma_2}{2} h_1^2 + \gamma_2 P_1 - 1)\right) = 0 \qquad (3.3)$$

The intersections of these curves with themselves and with the axis $\eta = 0$ from many regions in the $\eta\xi$-plane (Fig. 1). From (3.1) and (3.2), it follows that detonation-type discontinuities are evolutionary in the regions *43* and *21*, and combustion-type discontinuities, in the regions *44, 33, 22, 11*. We now consider the properties of the discontinuities corresponding to the different regions.

4. In the region 43^+ we have discontinuities with energy release (Fig. 1), at which the following inequalities hold:

$$a_{1+}^2 < V_1^2(\eta, 0) < V_1^2(\eta, q), \qquad V_2^2(\eta, 0) < V_2^2(\eta, q) \leqslant a_{2+}^2$$

We shall call these discontinuities super-Alfvenic detonations. The jumps in all the quantities at these discontinuities are positive, and $V_1^2 > V_2^2$. At these discontinuities, the zones of reaction follow fast shocks. Since across a reaction zone and shock wave, the mass flow is constant, then processes occurring there will correspond in the $\eta\xi$-plane to motion along the hyperbola

$$\xi\,(\eta + 1) = \xi_0 = \text{const}$$

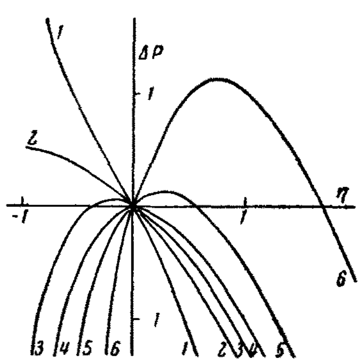

Fig. 3.

From the relations (2.5), (2.6) and (3.3), we obtain, along these hyperbolas

$$(\gamma_2 - 1)\,\Theta_2 dS_2 = dq = \frac{\varphi(\eta, \xi)}{(\xi_0 + 1)^2\,\xi}\,d\xi \qquad (4.1)$$

From this, it follows that for motion along the hyperbolas upward from the shock adiabat $\xi = \xi(\eta, 0)$, the entropy and the released energy q increase and assume maximums on the Jouguet curve for $V_2 = a_{2+}$, or, in other words, the above-mentioned hyperbolas and the curves $\xi = \xi(\eta, q)$ are tangent to each other on the Jouguet curve.

Furthermore, for motion along the hyperbolas with increasing q, the jumps in density, total pressure, and magnetic field all decrease toward the value at the Jouguet point, and the velocity V_2 increases toward the value a_{2+}. The jump in the gasdynamic pressure, as a function of the initial state and mass flow, either increases monotonically, or first decreases and then increases, to the value at the Jouguet point.

Figure 3 shows graphically the change in gasdynamic pressure at the discontinuities for constant mass flow. The values $0 < V_1^2 < 1$, $V_1^2 = 1$, $1 < V_1^2 < 1 + h_1^2$, $V_1^2 = h_1^2 + 1$, and $V_1^2 > h_1^2 + 1$ correspond to curves *1-6* in Fig. 3, respectively. Figure 4 shows graphically the total pressure for different q on the fast super-Alfvenic discontinuities. The conditions $V_1^2 = a_{1+}^2$, $V_2^2 = a_{2+}^2$ correspond to curves *1* and *2*; $q = 0$ corresponds to curve *3*.

The temperature jump $\Delta\theta$ for $\xi_0 + 1$, near a_{1+}^2, decreases monotonically, while in the opposite case, it first increases and then decreases, remaining positive.

In the region 43^- we have (Fig. 1) discontinuities of the detonation type with energy absorption ($q < 0$), at which the following inequalities hold

$$a_{1+}^2 < V_1^2(\eta, q) < V_1^2(\eta, 0), \qquad 1 < V_2^2(\eta, q) < V_2^2(\eta, 0)$$

We call these super-Alfvenic ionizations. In these waves, the ionization zone follows the fast shock wave.

The jumps in total pressure, density, magnetic field, gasdynamic pressure (for a perfect gas), and the dimensionless temperature are all positive, and $V_1^2 > V_2^2$. From (4.1), it follows that moving along the hyperbola $\xi(\eta + 1) = \xi_0$ downward from the shock adiabat $\xi = \xi(\eta, 0)$ (i.e. in the ionization zone), the quantities S_2 and q monotonically decrease.

The temperature jump $\Delta\theta$ increases monotonically for $a_{1+}^2 - 1 < \xi_0 < \xi_{01}$, first increases and then decreases for $\xi_{01} < \xi_0 < \xi_{02}$, and monotonically decreases for $\xi_{02} < \xi_0$. The jumps in total pressure, density, and magnetic field monotonically increase until either the entropy becomes equal to the value before the discontinuity, or the temperature reaches the critical value, i.e. that temperature for which ionization is impossible (Fig. 4).

In region (21^+) we have (Fig. 1) discontinuities of the detonation type with energy release, for which the following inequalities hold:

$$a_{1-}^2 < V_1^2(\eta, 0) < V_1^2(\eta, q) < 1, \qquad V_2^2(\eta, 0) < V_2^2(\eta, q) \leqslant a_{2-}^2$$

We call these discontinuities sub-Alfvenic detonations. In these waves, the reaction zone follows a slow shock wave.

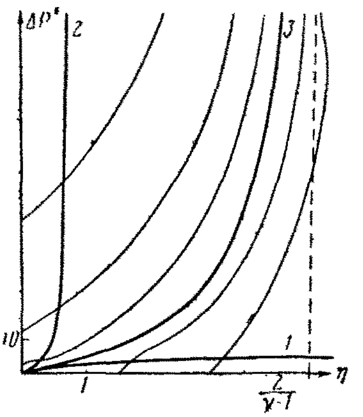

Fig. 4.

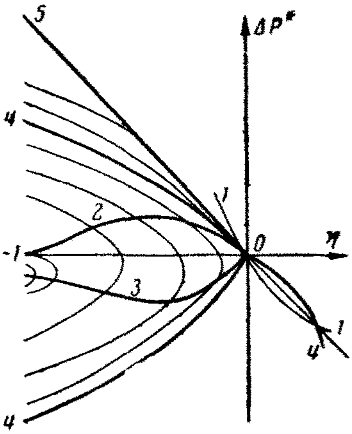

Fig. 5.

From relations (2.2) and (2.4), we obtain for these discontinuities

$$\Delta R > 0, \quad \Delta P > 0, \quad \Delta P^* > 0, \quad \Delta\theta > 0, \quad -h_1 < \Delta h < 0, \quad V_2 < V_1$$

From (4.1), it follows that moving along the hyperbola $\xi(\eta + 1) = \xi_0$ upward from the line $\xi = \xi(\eta, 0)$, which corresponds to the reaction zone, there results an increase in the parameter q and the entropy S_2 toward the maximum values, assumed on the Jouguet curve corresponding to $V_2 = a_{2-}$. Thus, at the Jouguet point, the hyperbola is tangent to the curves $\xi = \xi(\eta, q)$.

The jumps in density, total pressure, gasdynamic pressure, as well as the absolute value of the jump in the magnetic field, all decrease monotonically toward the values at the Jouguet point, and V_2 increases toward a_{2-} (Figs. 3, 5).

Figure 5 shows graphically the changes in total pressure at slow super-Alfvenic and sub-Alfvenic discontinuities. The states $V_1^2 = a_{1-}^2$, $V_2^2 = a_{2-}^2$, $V_{2+}^2 = a_{2+}^2$, $V_2 = 0$ correspond to the curves *1, 2, 3, 5*. The value $q = 0$ corresponds to curve *4*.

The temperature jump $\Delta\theta$ as a function of the initial parameters and the mass flow across the discontinuity may either increase monotonically, or first increase and then decrease.

We observe that sub-Alfvenic detonations occur only when the parameters h_1^2, p_1, γ_2 and q satisfy one of the following pairs of inequalities:

$$q \leqslant \frac{1}{2} h_1^2, \qquad \gamma_2\left(\frac{h_1^2}{2} + P_1\right) \geqslant 1 \tag{4.2}$$

$$\begin{gathered} q \leqslant \frac{1}{2} h_1^2 + \frac{1}{2(\gamma_2+1)}\left[\gamma_2\left(\frac{1}{2} h_1^2 + P_1\right) - 1\right]^2 \\ 0 < \gamma_2\left(\frac{h_1^2}{2} + P_1\right) < \sqrt{1-(\gamma_2^2-1)h_1^2} \end{gathered} \tag{4.3}$$

$$q \leqslant q^*(h_1^2, P_1, \gamma_2), \qquad \sqrt{1-(\gamma_2^2-1)h_1^2} < \gamma_2\left(\frac{1}{2} h_1^2 + P_1\right) < 1 \tag{4.4}$$

where q^* is the solution of the system (cf. 2.5).

$$f(\eta, \xi; h_1^2, P_1, \gamma_2, q) = 0$$

$$\frac{\partial}{\partial\eta} f(\eta, \xi, h_1^2, P_1, \gamma_2, q) = 0, \qquad \frac{\partial}{\partial\xi} f(\eta, \xi, h_1^2, P_1, \gamma_2, q) = 0$$

Inequalities (4.2) to (4.4) indicate that for fixed h_1, P_1, γ_2, the energy release is bounded. From inequality (4.2), we have

$$\rho_1 (C_1 - C_2)(\gamma_1 - 1) < H_{t1}^2 / 8\pi \qquad \text{for } \gamma_2 = \gamma_1$$

It is interesting that when inequality (4.4) holds and

$$\frac{1}{2} h_1^2 + \frac{\left[\gamma_2\left(\frac{1}{2} h_1^2 + P_1\right) - 1\right]^2}{2(\gamma_2+1)} < q$$

holds also, there exist on the detonation adiabat $\xi = \xi(\eta, q)$ two Jouguet points, corresponding to different shock speeds.

In the region $2I^-$, we have (Fig. 1) detonation-type discontinuities with energy absorption, for which the following inequalities obtain

$$a_{1-}^2 < V_1^2(\eta, q) < V_1(\eta, 0) < 1, \qquad V_2^2(\eta, q) < V_2^2(\eta, 0) \leqslant a_{2-}^2$$

We shall call this type of discontinuity sub-Alfvenic ionizations. In these, the ionization region follows a slow shock. For sub-Alfvenic ionizations, the following inequalities hold:

$$\Delta R > 0, \qquad \tfrac{1}{2} h_1^2 + 1 > \Delta P > 0, \qquad 1 > \Delta P^* > 0, \qquad V_1^2 > V_2^2$$

The temperature jump $\Delta\Theta$ may be positive (for small $|q|$) or negative. Moving along the hyperbola $\xi(\eta + 1) = \xi_0$ downward from the line

$\xi = \xi(\eta, 0)$, which corresponds to the ionization zone, the quantities S_2 and q decrease according to (4.1), and so does the velocity V_2 by (2.2). The temperature jump $\Delta\Theta$ for $\xi_0 + 1$ close to a_{1-}^2 first increases and then decreases, while in the converse case, it decreases monotonically.

The jumps in density, total and gasdynamic pressures, and the absolute value of the jump in magnetic field, all increase toward the values at which either the entropy becomes equal to the initial value, or the temperature equals the critical temperature.

For the discontinuities enumerated above, in addition to the quantities h_1^2, p_1, q_1, γ_2, it is necessary to give one of the two shock-strength parameters ξ or η. Then the calculation of the shock reduces to solving a quadratic equation (2.5), and all the quantities behind the discontinuity as well as the shock speed are given by relations (2.2), (2.4).

5. In region 44^+ we have (Fig. 1) combustion-type discontinuities with energy release, at which $a_1^{\ 2} < V_1^{\ 2}$ and $a_2^{\ 2} \leqslant V_2^{\ 2}$.

We shall call these fast super-Alfvenic combustion. This type of combustion may take place, for example, in thermonuclear reactions, when fusion causes the heating. The jumps in all the quantities at these fronts are positive, and $V_1^2 > V_2^2$.

In the region 22^+ we have (Fig. 1) combustion-type discontinuities, at which the inequalities $a_{1-}^{\ 2} < V_1^{\ 2} \leqslant 1$, $a_{2-}^{\ 2} \leqslant V_2^{\ 2} \leqslant 1$ obtain. We shall call these fast sub-Alfvenic combustion. On these discontinuities, we have the following inequalities:

$$\Delta R > 0,\quad \Delta P > 0,\quad \Delta P^* > 0,\quad \Delta\Theta > 0,\quad 0 > \Delta h > -h_1,\quad V_1^2 > V_2^2$$

Fast sub-Alfvenic combustion occurs for exactly the same values of the parameters h_1^2, p_1, γ_2, q as in sub-Alfvenic detonations (cf. (4.2), (4.3), (4.4)). Here also, for parameters satisfying the inequalities (4.4) and (4.5), there will be two distinct Jouguet points on the curve $\xi = \xi(\eta, q)$ corresponding to two different wave speeds, and at which $V_2 = a_{2-}$.

In the last two cases, for given reaction speeds V_1, i.e. moving along the hyperbola $(\eta + 1)\xi = \xi_0$ in the $\eta\xi$-plane from the initial state (axis $\eta = 0$), the entropy and released energy will both increase toward the maximum values assumed at the Jouguet points. Moreover, the velocity V_2 will decrease from the value V_1 toward a_{2+} or a_{2-}, for super- or sub-Alfvenic fronts, respectively. Jumps in density, total pressure, gas-dynamic pressure, and the absolute value of the jump in magnetic field, will also increase (Figs. 3-5).

In region *33*$^+$ we have (Fig. 1) discontinuities with energy release, at which the following inequalities hold:

$$1 \leqslant V_1^2 < a_{1+}^2, \qquad 1 \leqslant V_2^2 \leqslant a_{2+}^2$$

We shall call these slow super-Alfvenic combustion. These become ordinary gasdynamic combustion with subsonic velocities as the magnetic field is reduced to zero. On these discontinuities

$$\Delta R < 0, \qquad \Delta P^* < 0, \qquad -h_1 < \Delta h < 0, \qquad V_1^2 < V_2^2$$

The jumps in gasdynamic pressure and temperature may be either positive or negative. Slow super-Alfvenic combustion is possible only when the quantities h_1^2, p_1, γ_2, q satisfy one of the following pairs of inequalities:

$$2q < h_1^2 + \frac{[\gamma_2(^1/_2 h_1^2 + P_1) - 1]^2}{\gamma_2 + 1}, \qquad \gamma_2\left(\frac{1}{2}h_1^2 + P_1\right) \geqslant 1$$
$$2q \leqslant h_1^2, \qquad \gamma_2\left(\tfrac{1}{2}h_1^2 + P_1\right) < 1$$

In region *11*$^+$ we have (Fig. 1) discontinuities of the combustion type with energy release, at which the inequalities hold:

$$V_1^2 < a^2_{1-}, \qquad V_2^2 \leqslant a_{2-}^2$$

We call these slow sub-Alfvenic combustion. On these discontinuities, the following inequalities obtain:

$$-P_1 < \Delta P < 0, \qquad -h_1 A > \Delta h > 0, \qquad A < \Delta P^* < 0, \qquad V_2^2 > V_1^2$$
$$\left(A = \frac{h_1^2 + 1 - \sqrt{(h_1^2+1)^2 + 2P_1 h_1^2}}{h_1^2}\right)$$

The temperature jump $\Delta\Theta$ may be positive or negative, and the quantity q may be arbitrary.

For sub-Alfvenic and super-Alfvenic slow combustion, for fixed reaction speed and increase in intensity (corresponding to moving along the hyperbola $\xi(\eta + 1) = \xi_0$ from $\eta = 0$), the entropy and released energy q will increase from zero to the maximum values at the Jouguet points, according to (4.1). Moreover, the velocity V_2 will increase toward a_{2+}, a_{2-} and sub-Alfvenic fronts respectively.

The absolute values of the jumps in density, total pressure, magnetic field, and gasdynamic pressure, will increase monotonically from zero to the values at the Jouguet point, for sub-Alfvenic discontinuities.

For slow super-Alfvenic combustion, in which the combustion speed is greater than the Alfven speed in the resulting state (i.e. $V_1^2 > h_1^2 + 1$), the jump in the gasdynamic pressure will be negative, and with increase in q, will monotonically decrease to the value at the Jouguet point. When the combustion speed is less than the Alfven speed, the pressure jump first increases from zero to a maximum value, and then decreases to the value at the Jouguet point, which may be either positive or negative (Fig. 3)

$$\max \Delta P = V_1^2 - 1 + \tfrac{1}{2} h_1^2 - \tfrac{3}{2} \sqrt[3]{(V_1^2 - 1)^2 h_1^2} \quad \text{for } \eta + 1 = \sqrt[3]{\frac{V_1^2 - 1}{h_1^2}}$$

The jump in the dimensionless temperature, in the cases of slow super-Alfvenic and sub-Alfvenic discontinuities, behave in the same way as the pressure.

For $V_1^2 > B_\pm$ (corresponding to super-Alfvenic and sub-Alfvenic discontinuities), $\Delta\Theta$ is negative and monotonically decreases with increase in q; for $V_1^2 < B_\pm$ the temperature jump $\Delta\Theta$ first increases, and then decreases to the value at the Jouguet point

$$B_\pm = \frac{h_1^2 + P_1 + 1 \pm \sqrt{(h_1^2 + P_1 + 1)^2 + 4P_1}}{2}$$

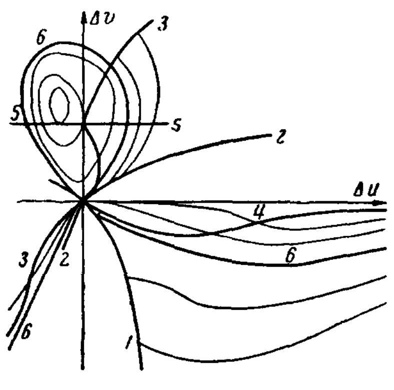

Fig. 6.

If, in a combustion type discontinuity, the reaction speed V_1 is known in addition to the quantities h_1^2, p_1, q, γ_2, then we get, from (2.2)

$$\xi = \frac{V_1^2 - 1}{\eta + 1}$$

Solving this jointly with (2.5), and using (2.2), we find all the quantities behind the front.

In Fig. 6, the thin lines represent curves connecting the jumps in the normal and tangential components of the gas velocity at the surface ($q \neq 0$). The states $V_1^2 = a_{1+}^2$, $V_1^2 = a_{1-}^2$, $V_2^2 = a_{2-}^2$, $V_2^2 = a_{2+}^2$, $h_2 = 0$ correspond to the curves *1*-*5*. Curve *6* corresponds to shock transition without energy release ($q = 0$).

Combustion type discontinuities, but with endothermic reactions, can be studied in a similar manner. We note that, according to (4.1), a decrease of entropy will occur with increase in $|q|$; such discontinuities in any given situation will occur when the entropy jump is positive, with the variation in γ considered.

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